AIAA JOURNAL
Vol. 36, No. 4, April 1998

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Optimization of Viscoelastic Compliant Walls for Transition Delay

K. S. Yeo*
National University of Singapore, 119260 Singapore

Introduction

IT is fairly well established by now that compliant walls are indeed capable of reducing the growth of disturbances in a laminar boundary layer and thus possess the potential for delaying the onset of laminar-turbulent transition. The theoretical and experimental evidence has been reviewed in recent publications by Carpenter[1] and Riley et al.[2] Compliant walls are susceptible to a wide variety of instabilities, which include the Tollmien–Schlichting instability (TSI), traveling-wave flutter (TWF), static divergence (SD), and other strong hybrid instabilities. These instabilities pose conflicting requirements on the properties of the compliant wall. The attainment of extensive transition delay is by no means a straightforward affair of simply coating a surface with anything that feels compliant to the touch. Significant stabilization generally requires the manipulation of a number of or all of the wall properties to simultaneously suppress all the instabilities.

The optimization of compliant wall properties to achieve large transition delay has been studied by Carpenter et al.[3] for many years, culminating in a recent paper by Dixon et al.,[4] which details procedures for optimizing single- and double-layer viscoelastic compliant walls. Some of the principles for obtaining good transition-delaying behavior have also been discussed by Yeo.[5,6] These works have focused on compliant layers with material density equal to that of the flow. The optimization of single-layer viscoelastic compliant walls is reconsidered in this paper without a prior assumption about density. An archetypal class of single-layer viscoelastic compliant wall that is capable theoretically of producing extensive delay of transition is presented.

Class of Optimized Compliant Walls

In optimizing compliant walls for transition delay, one needs to be concerned only with the three primary instabilities: the TSI, the TWF, and the SD. Strong hybrid instabilities, arising from the interaction of the three primary types, may occur on some highly compliant walls, but they are usually preceded by the occurrence of one or more of the primary instabilities.

The principle underlying the derivation of the present class of optimized walls is the stability criterion of Yeo and Dowling[7] for inviscid thin shear flow over passive compliant walls. The criterion, which was derived from generalized variational-theoretic considerations, is applicable to a wide range of passive compliant walls. Here, it is applied to the class of compliant walls obtained by attaching a single uniformly thick layer of an incompressible homogeneous isotropic viscoelastic material onto a rigid base; see Fig. 1. For such a wall, Yeo and Dowling have shown that the flow is linearly stable when

$$\underset{\text{static criterion,}}{\{2G,}\qquad \underset{\text{free-wave criterion}}{0.9126(G/\rho)\}} \quad > 1.0 \tag{1}$$

Here G and ρ denote the elastic shear modulus and the density of the compliant layer, respectively. These quantities are assumed to have been nondimensionalized with respect to freestream velocity U_∞ and flow density ρ_f.

Criterion (1) comprises two subcriteria that offer safeguards separately against the onset of SD (static criterion) and TWF (free-wave criterion). In its incipient state, the SD assumes the form of a static or slowly moving wave on the surface of the wall, whereas the TWF is a traveling-wave instability. The inviscid criterion (1) tends to be slightly conservative in practice. Its use therefore offers some margin of safety, which is desirable from a practical viewpoint. It is pertinent to note that criterion (1) has no dependence on the thickness of the layer h and level of material damping in the wall. Later, we shall see how a highly optimized class of transition-delaying single-layer walls may be derived from criterion (1) and a consideration of the well-known behavior of the TSI.

Criterion (1) can always be satisfied if the shear modulus G is sufficiently large. However, a large G would result in a highly stiff compliant wall, which suffers from strong TSI. A small G is desirable for suppressing the TSI. Thus, the shear modulus G should be kept as small as possible without violating criterion (1). Clearly the static criterion would be marginally satisfied if we take $G = 0.5$. This fixes the value of G for the free-wave criterion. The free-wave criterion will be satisfied if the material density ρ of the compliant layer is sufficiently small. However, reduction in ρ is also known to destabilize the TSI. Hence ρ should be as large as possible to give a highly subdued TSI regime without violating the free-wave criterion. For $G = 0.5$, the largest value of ρ we may use without violating the free-wave criterion is 0.4563. Thus, we have arrived at a class of optimized compliant walls that have

$$G = 0.5, \qquad \rho = 0.4563 \tag{2}$$

The preceding discussion indicates that these wall properties would allow us to suppress the SD and TWF with minimal adverse effects on the TSI. Because the suppression of the TWF and SD instabilities based on the wall properties (2) is not dependent on the thickness h and the damping quality of the compliant layer, we may now freely select these to further optimize the layer with respect to the stabilization of the TSI.

Increase in thickness h of the compliant layer makes the wall more compliant. Thus a thick compliant layer will enhance the stability of the flow with regard to the TSI. However, the studies of

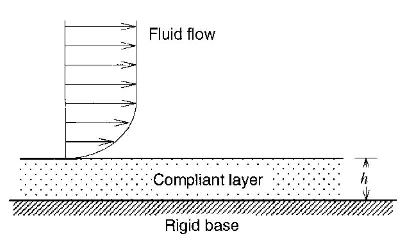

Fig. 1 Flow over a compliant layer backed by a rigid base.

Received Sept. 9, 1996; revision received Aug. 15, 1997; accepted for publication Nov. 17, 1997.

*Associate Professor, Department of Mechanical and Production Engineering, Kent Ridge Crescent. E-mail: mpeyeoks@nus.edu.sg.

Table 1 Variations of R_δ^{tr}, R_x^{tr}, and TDF with the thickness h of the compliant layer

Thickness	$N = 8.3$			$N = 7.0$		
h	$R_\delta^{tr} \times 10^{-3}$	$R_x^{tr} \times 10^{-6}$	TDF	$R_\delta^{tr} \times 10^{-3}$	$R_x^{tr} \times 10^{-6}$	TDF
0 (rigid wall)	2.9	2.84	1.0	2.57	2.23	1.0
1.0	3.63	4.45	1.57	3.3	3.68	1.65
5.0	5.84	11.50	4.05	5.33	9.59	4.30
10.0	6.85	15.85	5.58	5.91	11.79	5.29
20.0	6.95	16.31	5.74	5.96	12.00	5.38

Yeo[5,6] have shown that marginal gain in TS stability diminishes with increase in h. Beyond a certain point, the incremental gain would not justify further increase in h. This naturally limits the thickness that one should use. The appropriate choice of thickness h will be determined from viscous flow calculations because the TSI is a viscosity-based instability. Because it is well known that material damping destabilizes the TSI, the optimized wall should also have minimum damping. Thus the class of optimized walls that we have arrived at would be a fairly thick layer wall with minimal material damping, material density $\rho = 0.4563$, and shear modulus $G = 0.5$. In dimensional terms, this means that the material density and elastic (storage) shear modulus of the compliant layer should be $0.4563\rho_f$ and $0.5\rho_f U_\infty^2$, respectively.

The condition $G = 0.5$ has also appeared in Ref. 4. However, their optimization presupposed a fixed material density of $\rho = 1.0$, and maximization of transition delay was then achieved through variations of layer thickness h, material damping, and shear modulus G. In contrast, our optimization begins with no presumption of wall parameter. The optimal compliant-layer parameters are derived in a single logical sequence: starting from the SD criterion, which yields $G = 0.5$; followed by the TWF criterion, which fixes the best value for $\rho = 0.4563$; and then the maximum stabilization of the TSI through the adoption of minimum wall damping and a thick compliant layer.

Viscous Flow/Orr–Sommerfeld Calculations

Boundary-layer stability calculations were carried out following Yeo[5,6] for layers with properties specified by Eq. (2) and nondimensional thickness ranging from $h = 1$ to 20 (thickness is nondimensionalized by the displacement thickness δ of the boundary layer at Reynolds number $R_\delta = U_\infty \delta / \nu$ of 2×10^4). For water of kinematic viscosity $\nu = 10^{-6}$ m^2/s and flowing with freestream speed of $U_\infty = 10$ m/s, these correspond to a thickness ranging from 2 to 40 mm. Material damping is provided by a Kelvin–Voigt model for shear deformation: $G^c = G - i\omega d$ being the complex shear modulus, where d is the damping coefficient and ω the frequency of the perturbation wave. Because no materials are perfectly elastic, a small $d = 0.0049$ was used throughout to simulate the response of highly elastic layers. At this low level of damping and relatively high G, SD simply does not exist.[8] Computations confirm the absence of TWF for all the walls. Increase in d will render the TWF more stable and but will progressively destabilize the TSI. For a given h, the best stability performance will therefore be achieved with the lowest possible material damping.

Transition Delay

The laminar-turbulent transition Reynolds number R_δ^{tr} is estimated here, as in Refs. 4 and 6, by applying the well-established e^N transition correlation rule of Smith and Gamberoni.[9] A value of $N = 8.3$ is used here, as Ref. 6. For this value of N, transition on a rigid wall would be expected to occur at $R_\delta^{tr} \approx 2.9 \times 10^3$, corresponding to the now classical experiments of Schubauer and Skramstad.[10] The associated transition Reynolds number based on the distance from the leading edge x is $R_x^{tr} = U_\infty x / \nu \approx 2.84 \times 10^6$. Larger values of N may be used for flows with very low levels of turbulence. A more conservative value of $N = 7$ has generally been used by Dixon et al. Figure 2 shows the maximum amplification envelope for the optimized wall with thickness $h = 10$. For $N = 8.3$, it has a transition Reynolds number R_δ^{tr} of approximately 6.85×10^3 ($R_x^{tr} \approx 15.85 \times 10^6$). The transition delay factor (TDF) defined by

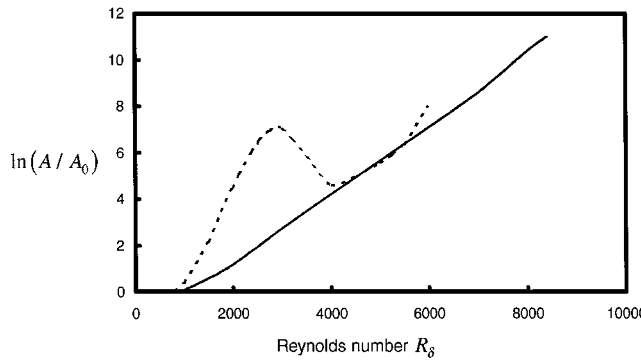

Fig. 2 Maximum amplification envelopes: ——, present optimized single-layer wall with $h = 10$, and – – –, optimized two-layer wall of Dixon et al.[4]

$$\text{TDF} = \frac{\{R_x^{tr}\}_{\text{compliant wall}}}{\{R_x^{tr}\}_{\text{rigid wall}}} \tag{3}$$

has a value of 5.58. The R_δ^{tr} is reduced to about 5.91×10^3 ($R_x^{tr} \approx 11.79 \times 10^6$) if a value of $N = 7$ is used instead, which yields a TDF of about 5.13 when compared with the corresponding rigid wall $R_\delta^{tr} \approx 2.57 \times 10^3$ ($R_x^{tr} \approx 2.23 \times 10^6$). The transition Reynolds numbers and the TDF for the present class of optimized walls with thickness $h = 1$, 5, 10, and 20 are summarized in Table 1 for logarithmic growth factors of 8.3 and 7.0.

For $N = 8.3$, doubling of thickness h from 10 to 20 produces only a modest 2.9% increase in the TDF. A layer of thickness of $h = 10$ may thus be regarded as being fairly optimal, in having captured much of the available stability gain that may be derived from this class of compliant walls. For $N = 7.0$, a compliant thickness of $h \approx 8$ would be fairly optimal.

The transition Reynolds numbers and TDF achieved by the present class of optimized compliant walls are larger than corresponding values for the optimized single-layer walls of Dixon et al.,[4] which have $\rho = 1$. The present values are in fact comparable to the optimized results of their more complex two-layer compliant walls, which have R_δ^{tr} ranging from 5.7×10^3 to 6×10^3 ($N = 7.0$). The maximum amplification envelope for an optimized two-layer wall of Dixon et al. is also depicted in Fig. 2. It has a sigmoid shape with a local maximum value close to 7.0 at $R_\delta \approx 3 \times 10^3$. Freestream turbulence can cause the exponent N to fall below 7.0. This could trigger a sudden fall of R_δ^{tr} from 6×10^3 to 3×10^3. The present optimized wall, with its linearly rising amplification envelope is expected, however, to respond to changes in freestream turbulence in a stable and progressive manner, with no sudden drop in R_δ^{tr}.

Works on compliant wall flow stabilization[4–6] to date have tended to focus almost exclusively on compliant materials with a density equal to that of the flow, i.e., $\rho = 1$. This is because suitable compliant materials have generally been assumed to be solid rubber-type polymeric compounds. These have density close to water, which is the fluid medium in which compliant surfaces are likely to be deployed. The present study shows that compliant materials with lower ρ possess highly superior stability characteristics. Small variation of ρ from the optimal value of 0.4563 is not expected to have unduly large adverse effect on stability: A decrease in ρ would actually improve TWF stability with a progressive destabilizing effect on the TSI, whereas the destabilization of TWF caused by an increase in ρ could be easily kept in check by increasing d slightly. To allow for possible inaccuracies in material properties, it is preferable to use the walls within a narrow velocity window just below the optimal conditions.

Materials suitable for the proposed walls are likely to be foam-based. These materials are lighter than water and possess a relatively high stiffness-to-weight ratio that renders them particularly effective against hydroelastic instabilities. The process of their manufacture influences their properties. They also offer the possibility of their properties being designed and manufactured to specification. Some variety may also offer very low damping because of their partially hollow nature (less material involved in straining). The bulk modulus of foam-based materials may theoretically be regulated by the pressure of the enclosed gas phase. With a view to the future development of materials technology, the construction of compliant coatings from foam-based polymeric materials cannot be ruled out. Navier's equations will be applicable to such materials if the cell size is much less than the wavelength of the perturbations.

The present study has focused on two-dimensional instability modes because these are known to be the most unstable for the TWF and SD. For the TSI, two-dimensional studies are still effective for comparing the relative stability performance of different isotropic compliant walls, although prediction on transition gain will be slightly reduced when three-dimensional modes are also considered.[11,12]

Conclusions

The present Note considers the optimization of single-layer viscoelastic compliant walls for delay of laminar-turbulent boundary-layer transition. The archetypal transition-delaying compliant layers should possess a nondimensional material density of 0.4563, elastic shear modulus of 0.5, and minimal material damping and should be sufficiently thick. The transition distances and transition delay factors obtained in the present study are comparable to the values reported by Dixon et al.[4] for their unit-density two-layer walls.

Acknowledgment

The results reported in this Note were obtained with the assistance of K. W. Lum.

References

[1]Carpenter, P. W., "Status of Transition Delay Using Compliant Walls," *Viscous Drag Reduction in Boundary Layers*, edited by D. M. Bushnell and J. N. Heffner, Vol. 123, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1990, pp. 79–113.

[2]Riley, J. J., Gad-el-Hak, M., and Metcalfe, R. W., "Compliant Coatings," *Annual Review of Fluid Mechanics*, Vol. 20, 1988, pp. 393–420.

[3]Carpenter, P. W., Lucey, A. D., and Dixon, A. E., "The Optimization of Compliant Walls for Drag Reduction," *Recent Developments in Turbulence Management*, edited by K. S. Choi, Kluwer, Dordrecht, The Netherlands, 1991, pp. 195–221.

[4]Dixon, A. E., Lucey, A. D., and Carpenter, P. W., "Optimization of Viscoelastic Compliant Walls for Transition Delay," *AIAA Journal*, Vol. 32, No. 2, 1994, pp. 256–267.

[5]Yeo, K. S., "The Stability of Flow over Flexible Surfaces," Ph.D. Thesis, Dept. of Engineering, Univ. of Cambridge, Cambridge, England, UK, Nov. 1986.

[6]Yeo, K. S., "The Stability of Boundary-Layer Flow over Single- and Multi-Layer Viscoelastic Walls," *Journal of Fluid Mechanics*, Vol. 196, 1988, pp. 359–408.

[7]Yeo, K. S., and Dowling, A. P., "The Stability of Inviscid Flow over Passive Compliant Walls," *Journal of Fluid Mechanics*, Vol. 183, 1987, pp. 265–292.

[8]Yeo, K. S., Khoo, B. C., and Zhao, H. Z., "The Absolute Instability of Boundary-Layer Flow over Viscoelastic Walls," *Theoretical and Computational Fluid Dynamics*, Vol. 8, 1996, pp. 237–252.

[9]Smith, A. M. O., and Gamberoni, H., "Transition, Pressure Gradient and Stability Theory," Douglas Aircraft Co., Rept. ES26388, Long Beach, CA, 1956.

[10]Schubauer, G. B., and Skramstad, H. K., "Laminar Boundary-Layer Oscillations and Transition on a Flat Plate," NACA Rept. 909, 1948.

[11]Yeo, K. S., "The Three-Dimensional Stability of Boundary-Layer Flow over Compliant Walls," *Journal of Fluid Mechanics*, Vol. 238, 1992, pp. 537–577.

[12]Joslin, R. D., Morris, P. J., and Carpenter, P. W., "The Role of Three-Dimensional Instabilities in Compliant-Wall Boundary-Layer Transition," *AIAA Journal*, Vol. 29, No. 10, 1991, pp. 1603–1610.

J. Kallinderis
Associate Editor

Turbulence Structure in the Spiral Wake Shed by a Lifting Wing

Joseph A. Miranda* and William J. Devenport†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24060

I. Introduction

THE wakes of large-aspect-ratio lifting wings are of great engineering and scientific interest. Much effort has been focused on understanding the tip vortices, but relatively little attention has been paid to the rest of the wake, specifically the axial wake of the wing boundary layers, and its development in the presence of the vortex. There have been several studies documenting the presence of this part of the wake and its evolving spiral form.[1–5] Of the most detail and of particular interest here is the work of Devenport et al.[1] Their measurements show the axial wake simultaneously decaying and rolling up around the vortex core, its turbulence structure evolving in the presence of lateral stretching, skewing, and lateral curvature. They show that, despite these complex and changing influences, the turbulence structure of this flow achieves an approximately self-similar state when normalized on the scale of the wake spiral and the axial velocity scale of the two-dimensional part of wake. This makes the flow of particular interest as an experimental and computational test case.

This Note summarizes a study[6] of a spiral wake similar to that of Devenport et al.[1] but in much greater detail. Our goal was to reveal precisely the effects of the different three-dimensional influences that act on the spiral wake and to compare its structure with the near-two-dimensional region found farther inboard. Both single- and two-point hot-wire measurements were made to this end. Single-point measurements are presented here; related two-point measurements and discussion relevant to aeroacoustic applications are dealt with in a companion paper.[7]

II. Apparatus and Instrumentation

The flow was generated in the $3 \times 2 \times 20$ ft test section of the Virginia Tech Subsonic Wind Tunnel using a rectangular planform, NACA 0012 half wing of 0.203-m chord c placed at 5-deg angle of attack. Flow in the empty test section is closely uniform, with a turbulence intensity of less than 0.3% (Ref. 8). A slight favorable streamwise pressure gradient $dC_p/dx = -0.96\%$/m is present because of boundary-layer growth. The wing was mounted at the midheight of the test section and extended two thirds of the way across its 3-ft width. The wing boundary layers were tripped using 0.5-mm-diam glass beads glued in a random pattern between the

Presented as Paper 96-0804 at the AIAA 34th Aerospace Sciences Meeting, Reno, NV, Jan. 15–18, 1996; received June 5, 1997; revision received Nov. 25, 1997; accepted for publication Dec. 11, 1997.

*Graduate Research Assistant, Department of Aerospace and Ocean Engineering; currently Mechanical Engineer, U.S. Naval Undersea Warfare Center, Newport, RI 02847. Associate Member AIAA.

†Associate Professor, Department of Aerospace and Ocean Engineering, 215 Randolph Hall. Senior Member AIAA.